II. "Dynamo-Electric Machines.—Preliminary Notice." By John Hopkinson, D.Sc., F.R.S., and Edward Hopkinson, D.Sc. Received April 3, 1886.

Omitting the inductive effects of the current in the armature itself, all the properties of a dynamo machine are most conveniently deduced from a statement of the relation between the magnetic field and the magnetising force required to produce that field. This relation given, it is easy to deduce what the result will be in all employments of the machine, also the result of varying the winding of the machine in armature or magnets. The magnetic field may be expressed algebraically as a function of the magnetising force, or more conveniently by a curve ("Proceedings of the Institution of Mechanical Engineers," April, 1879, p. 246). Amongst the empirical formulæ which have been proposed to express the electromotive force of dynamo machines in terms of the currents around the magnets, we may mention that known as Fröhlich's, where $E = \frac{ac}{1+bc}$, E being the electromotive force of the machine at a given speed, c the exciting current, and a and b constants. For some machines this hyperbola is said to express observed results fairly accurately. In our experience it does not sufficiently approximate to a straight line in the part of the curve near the origin, and gives too high results for large values of c.

One purpose of the present investigation is to give an approximately complete construction of the characteristic curve of a dynamo of given form from the ordinary laws of electromagnetism and the known properties of iron. Let n be the number of convolutions on the magnets, c the current round the magnets, l_1 the mean length of the lines of force in the iron of the armature, A_1 the area of section of iron in the armature, l_2 the distance from iron of armature to iron of pole pieces, A_2 the area of the magnetic field in which the wires move corrected for its extension round the edge of the pole pieces, l_3 the total length of the magnet cores, A_3 the area of the magnet cores, l_4 the mean length of lines of force in the yoke connecting the magnet limbs in machines of the type on which we have principally experimented, A_4 the area of section of the yoke, l_5 the mean length of the lines of force in each pole piece, A_5 the mean area of section of pole piece, I the total induction through the armature, when no current passes in the armature, and νI the total induction in the magnet cores; and finally let the relation between the magnetic force (α) and induction (a) (*vide* Thomson, "Electrostatics and Magnetism,"

p. 397, and Maxwell, "Treatise on Electricity and Magnetism," vol. ii, p. 24) be represented by the equation $\alpha = f(a)$, then the characteristic curve is—

$$4\pi nc = l_1 f\left(\frac{\mathrm{I}}{\mathrm{A}_1}\right) + 2l_2 \frac{\mathrm{I}}{\mathrm{A}_2} + l_3 f\left(\frac{\nu \mathrm{I}}{\mathrm{A}_3}\right) + l_4 f\left(\frac{\nu \mathrm{I}}{\mathrm{A}_4}\right) + 2l_5 f\left(\frac{\mathrm{I}_5}{\mathrm{A}_5}\right).$$

If the relation between α and a be given in the form of a curve, this formula indicates at once a perfectly simple graphical construction for the characteristic. Taking the curve of magnetisation determined by one of us for wrought iron, and constructing a characteristic in this way, we have obtained a theoretical curve which agrees over a long range with the actual results of observation on a dynamo machine more closely than any empirical formula with which we are acquainted.

To determine ν, a wire was taken once round the middle of one magnet and connected to a ballistic galvanometer, a known current was then either suddenly passed round the magnets or short circuited, the elongation of the galvanometer being noted. A similar observation was made with the same current, the galvanometer being connected to a single convolution of the armature in the plane of commutation. The ratio of the two elongations is the value of ν.

The distribution of the waste field $(\nu-1)\,\mathrm{I}$ was roughly ascertained in a similar manner.

The currents in the fixed coils round the magnets are not the only magnetising forces applied in a dynamo machine. The currents in the moving coils of the armature have also their effect upon the resultant field. In well-constructed machines the effect of the latter is reduced to a minimum, but it can be by no means neglected. This introduces a second independent variable, viz., C, the current in the armature. The effect of the current in the armature depends upon the lead given to the brushes. Denote this by λ, which we may also regard as an independent variable, as it is subject to arbitrary adjustment.

If $\mathrm{I} = \mathrm{F}(4\pi nc)$ be the characteristic curve when no current passes through the armature, then

$$\mathrm{I} + \frac{\nu-1}{\nu}\, 4\lambda m \mathrm{C}\, \frac{\mathrm{A}_2}{l_2} = \mathrm{F}\left(4\pi nc - \frac{4\lambda m \mathrm{C}}{\nu}\right),$$

where m is the number of convolutions in the armature. Here we omit the comparatively unimportant portion of the magnetic force in the core of the armature and the pole pieces. From this formula it is not difficult to deduce a geometrical construction for the characteristic surface (*vide* "Practical Applications of Electricity," Lectures delivered at the Institute of Civil Engineers, 1882–83, p. 98). The

equation may be thus expressed in words, if λ be such that the coils at commutation embrace the whole or nearly the whole induction. The effect of the current in the armature upon the difference of potential between the brushes of any machine, is the same as that of an addition to the resistance of the armature proportional to the lead of the brushes, and to the ratio of the waste field to the total field, combined with that of taking the main current $\frac{m\lambda}{\nu\pi}$ times round the magnets in a direction opposite to the current c. Many consequences can be deduced, of which we may notice the following:—In a series wound dynamo C is equal to c, and if c be increased beyond a certain point, I must attain a maximum and then diminish; this has been frequently observed. We now see that it depends upon the existence of a waste field. Secondly, let the coils of the magnets be entirely disconnected, and let λ be the negative: if the armature be short circuited through a small resistance and be run at a sufficient speed, a large current may be produced in the armature. This latter deduction we have verified by direct experiment.

The efficiency of the type of dynamo machine upon which the experiments before indicated have been made, has been accurately determined by the device of coupling two similar machines, both mechanically and electrically, so that one should act as a generator of electricity, driving the other electrically, whilst the latter acted as a motor driving the former mechanically; the loss of power required to keep the whole combination in movement being determined by direct dynamometric measurement, and the power passing electrically from the one machine to the other being measured by ordinary electrical appliances.

The whole of the experiments were carried out at the works of Messrs. Mather and Platt, to whom we are indebted for the exceptional opportunities we have enjoyed of putting theoretical conclusions to the test of experiment on an engineering scale.

The Society then adjourned over the Easter Recess to Thursday, May 6th.